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Optimal-Suboptimal Synthesis and Design of Flotation Circuits

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Abstract

The integrated approach of process synthesis and design has been applied to determine the optimal-suboptimal configuration and design parameters of a flotation circuit for separation of mineral species. Mean residence time of particulate species in each cell is the design parameter to be optimized, which along with the optimal structure is extracted simultaneously from a generalized circuit by direct search. The validity of the method has been demonstrated by comparing results obtained by using this method with real optimal structure and parameter values obtained by optimizing all feasible structures, enumerated one at a time, for the case of a two cell circuit using two different objective functions of recovery, grade, and profit. The method is then extended to more realistic four cell circuit and feed comprising valuable mineral, gangue, and middlings.

INTRODUCTION

Froth flotation of suspended particulate solids by means of air bubbles is one of the most important methods available at present for separation of valuable mineral ores from associated gangue impurities. In a single compartment continuous flotation cell there is one feed stream of raw ground ore slurry and two exit streams, comprised of mineral-rich floated concentrate and gangue-rich tailings. The aim of the flotation process is to achieve maximum recovery (ratio of solid flow rates in concentrate to that in feed) and highest grade (fraction of valuable mineral in the concentrate). Due to many reasons the separation is seldom complete, and in order to improve the process efficiency it is a normal practice to employ a number of interconnected flotation cells or banks of flotation cells. Feed is introduced in the rougher cell where a crude separation is effected. For improvement of its grade, the concentrate is then refloated in one or more cleaner and recleaner cells. The tailings from the rougher stage is refloated in a scavenger cell in order to extract, as far as possible, the residual mineral from the gangue before it leaves the circuit.

A typical flotation circuit may process many thousands of tons of ore per annum. Even a marginal improvement in its process efficiency can have a significant economic impact. More so, as mineral-rich ores are being rapidly depleted and it has become necessary to treat progressively lower quality ore, with attendant increase in the processing cost. In this preliminary report, we have investigated the feasibility of synthesizing and designing optimal and suboptimal multicell flotation circuits using a number of appropriate objective functions of grade, recovery, and profit.

STATEMENT OF THE PROBLEM

For illustration purpose, consider two interconnected cells. Figure 1 shows six possible intuitively meaningful configurations in which, for the time being, split streams are excluded. In the trial and error approach the problem is broken into two steps. First, all possible configurations are enumerated; second, the best design parameters are determined for each structure and the optimum configuration is selected by comparing the objective functions. In general, a flotation circuit is comprised of many cells, performing rougher, cleaner, and scavenger functions, and it is quite tedious to enumerate all meaningful structures. Moreover, there is no a priori justification for ignoring split streams, and the enumeration problem thus becomes even more complex. Umeda, Ichikawa, and Hirai (*1*) have demonstrated an integral approach to the problem in which the enumerations are totally circumvented and the optimal structure and design parameters are extracted simultaneously from a generalized circuit by direct search. We have used this method to solve the linkage (including split streams) problem and to determine the optimal residence time distribution of particles in each cell. The residence time distribution, as a design parameter, is a natural engineering choice since, as pointed out by Woodburn et al. (*2*), only the holding time is at present capable of being included into a quantitative description of the flotation kinetics. These authors have determined optimal mean residence times in perfectly mixed cells for a given circuit configuration.

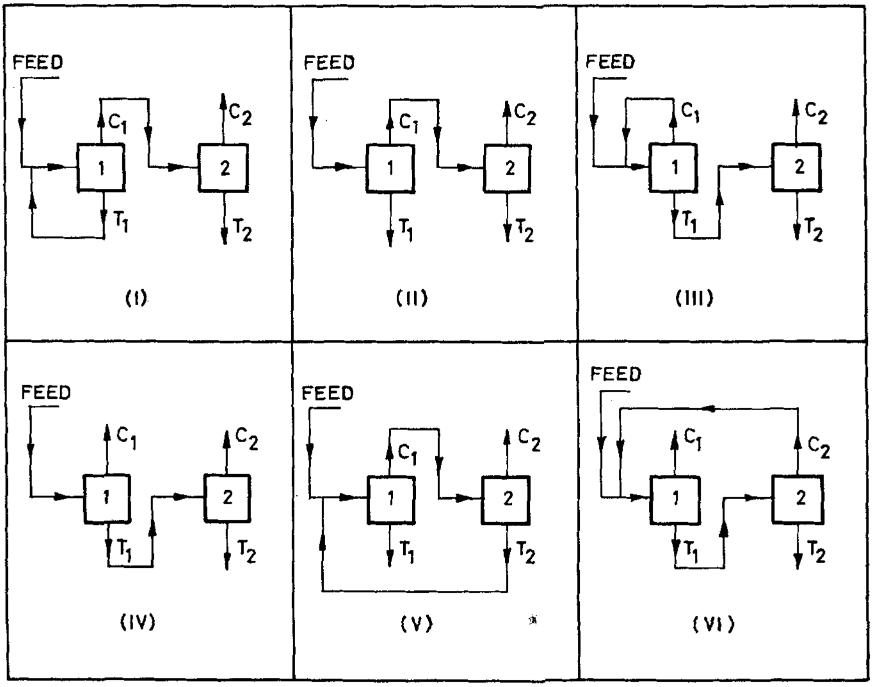

FIG. 1. Six possible configurations for a circuit with two flotation cells.

FLOTATION KINETIC MODEL

Considerable experimental and theoretical work in literature points to a first-order rate of flotation of particulate solids with a distributed flotation rate constant (*3–8*). It is also known that the cell, for all practical purposes, behaves like a perfect mixer (*2, 5, 6, 9–11*), but the mean residence time is a function of particle size and density. Thus if $M_F(K)$ is mass flow rate of particles of flotation rate constant K to the cell and λ is the mean residence time, the mass flow rates in concentrate and tailings streams at steady state are (*12*)

$$C(K) = M_F(K)\left[\frac{K\lambda}{1 + K\lambda}\right] \tag{1}$$

and

$$T(K) = M_F(K)\left[\frac{1}{1 + K\lambda}\right] \tag{2}$$

Using these relationships we can construct a steady state mathematical model for n-particulate species of the same size and density in an m-cell flotation circuit. Let the specific flotation rate constant of the jth species be K_j and let w_j be its fractional valuable mineral content, and λ_i be the mean residence time in ith cell. In the generalized configuration, the feed is split into m streams and fed to each cell. Similarly, each concentrate and tailings stream is split into $(m + 1)$ streams and connected as feed to to every other cell as well as short circuited back to the cell from which it originated. The $(m + 1)$-th split stream leaves the circuit as concentrate or tailings output. The generalized configuration for two cells is shown in Fig. 2. Consider now the ith cell. The total feed flow rate of jth species to this cell is

$$F_{j,i} = M_{F,j}\,\delta_{Fi} + \sum_{k=1}^{m} T_{j,k}\delta_{ki} + \sum_{k=1}^{m} C_{j,k}\beta_{ki} \tag{3}$$

where $M_{F,j}$ is the flow rate of the jth component in new feed; δ_{Fi} is the fraction of this feed going to the ith cell, $T_{j,k}$ is the tailings flow rate of the

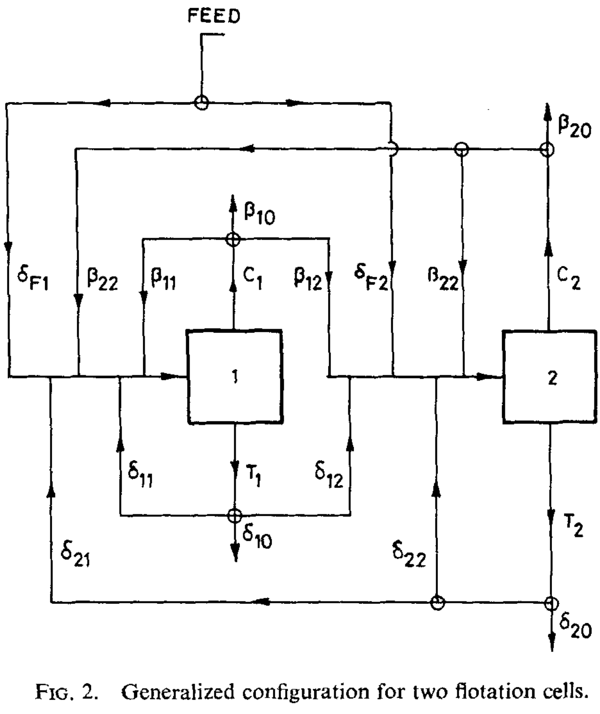

FIG. 2. Generalized configuration for two flotation cells.

jth species from the kth cell, δ_{ki} is the split fraction of this flow going to the ith cell. $C_{j,k}$ is the concentrate flow rate of the jth species originating from the kth cell, and β_{ki} is the split fraction of concentrate flow from the kth to the ith cell. Let

$$\alpha_{ji} = \frac{1}{1 + K_j\lambda_i} \tag{4}$$

The flow rates of the jth species from the ith cell in concentrate and tailings streams are

$$C_{j,i} = F_{j,i}\,[1 - \alpha_{ji}] \tag{5}$$

and

$$T_{j,i} = F_{j,i}\,\alpha_{ji} \tag{6}$$

Hence

$$C_{j,i} = T_{j,i}\left[\frac{1 - \alpha_{ji}}{\alpha_{ji}}\right] \tag{7}$$

Substitution in Eq. (3) gives

$$F_{j,i} = M_{F,j}\,\delta_{Fi} + \sum_{k=1}^{m} T_{j,k}\left\{\left[\frac{1 - \alpha_{jk}}{\alpha_{jk}}\right]\beta_{ki} + \delta_{ki}\right\} \tag{8}$$

Substitution in Eq. (6) gives

$$T_{j,i} = M_{F,j}\delta_{Fi}\alpha_{ji} + \alpha_{ji}\sum_{k=1}^{m} T_{j,k}\left\{\left[\frac{1 - \alpha_{jk}}{\alpha_{jk}}\right]\beta_{ki} + \delta_{ki}\right\}, \quad \begin{array}{l} j = 1, 2, \ldots, n \\ i = 1, 2, \ldots, m \end{array} \tag{9}$$

These $(n + m)$ linear algebraic equations can be solved for $T_{j,i}$ in terms of linkage and design parameters. The formal solution is

$$T_{j,i} = \phi_{j,i}\,(M_{F,j}, \delta, \beta, \alpha) \tag{10}$$

Similarly from Eq. (7)

$$C_{j,i} = \phi_{j,i}\left[\frac{1 - \alpha_{ji}}{\alpha_{ji}}\right] \tag{11}$$

Let δ_{i0} and β_{i0} be the fractions of tailings and concentrate from the ith cell, respectively, leaving the circuit, then the overall percentage recovery is

$$M_c = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} C_{j,i}\beta_{i0}}{\sum_{j=1}^{n} M_{F,j}} \times 100 \tag{12}$$

The grade is

$$G = \frac{\sum_{i=1}^{m} \sum_{j=1}^{n} C_{j,i} \beta_{i0} w_j}{\sum_{i=1}^{m} \sum_{j=1}^{n} C_{j,i} \beta_{i0}} \times 100 \tag{13}$$

CONSTRAINTS

The problem is now reduced to finding the best possible values of δ, β, α such that an objective function of M_c and/or G is optimized, given $M_{F,j}$, K_j ($j = 1, 2, \ldots, n$), and the number of cells m, and subject to the following constraints. From considerations of mass balance

$$\sum_{i=1}^{m} \delta_{Fi} = 1 \tag{14}$$

$$\sum_{i=1}^{m} \delta_{ki} + \delta_{k0} = 1 \tag{15}$$

$$\sum_{i=1}^{m} \beta_{ki} + \beta_{k0} = 1 \tag{16}$$

$$0 \leqq (\delta, \beta) \leqq 1 \tag{17}$$

Moreover

$$0 \leqq \alpha_{ji} \leqq 1 \tag{18}$$

Additional constraints must be incorporated if the optimal circuit is to be realizable from an engineering point of view, and physically meaningful. Thus it is known that for reasons of overloading of bubbles and loss of flotation selectivity, the pulp density d_p (percent mass of solid in slurry) should not exceed a prescribed value $\bar{d}_p$ in any cell. By definition of the mean residence time, the hold-up of solids in the ith cell is

$$H_i = \lambda_i \sum_{j=1}^{n} T_{ji} \tag{19}$$

Therefore, the lower bound on volume of the ith cell is

$$V_i \geqslant H_i \left[\frac{1}{d} + \frac{100 - \bar{d}_p}{\bar{d}_p} \right] \tag{20}$$

where d is solid density. Moreover, the overall size of the plant is also restricted to some reasonable volume V of all cells combined, hence

$$V = \sum_{i=1}^{m} V_i \tag{21}$$

For convenience in computation, personal judgment is used to impose a quite liberal upper constraint $\bar{\lambda}$ on λ, i.e.,

$$0 \leqq \lambda_i \leqq \bar{\lambda}, \qquad i = 1, 2, \ldots, m \tag{22}$$

To be acceptable, it is necessary that the grade of the concentrate does not drop below a minimum level $\bar{G}$, which gives rise to

$$G \geqslant \bar{G} \tag{23}$$

An obvious objective is to maximize recovery given in Eq. (12), subject to the above constraints. In view of the fact that as $K_1 > K_2, \ldots, > K_n$ in general, $w_1 > w_2, \ldots, > w_n$, we may infer from the physical nature of the problem that the optimum lies on the boundary of constraint $G = \bar{G}$. This, however, may not necessarily be true for more complicated objective functions as illustrated subsequently.

COMPUTATIONAL ASPECTS

The integral approach used here requires search through a rather large parameter space. This gives rise to well-known difficulties in the matter of the slow rate of convergence and false optimum. An allied problem is of deciding whether a split stream is, in fact, missing or not when linkage parameters δ and β are very near 1 or zero. On the other hand, realization of a large number of suboptima is not necessarily a disadvantage, since some configurations may turn out to be simpler to design and operate than the global optimum structure with only a marginal loss in performance.

The set of algebraic equations defined by Eq. (9) was solved by the Gauss Jordon technique (*13*). A systematic evaluation of the "best" optimization algorithm was beyond the scope of this investigation. However, modified complex (*14*, *15*) and random search (*16–18*) methods were employed. Success with the first algorithm, used by Umeda et al. (*1*) also, was found to be highly sensitive to the initial guess; the latter method, although slow, was quite reliable. In every case a number of different initial guesses were tried in order to assure, as far as possible, a true optimum. The optimization was carried out in two or more stages. In the course of computation it was observed that eventually the improvement in the objective function became rather slow. The search was stopped and the linkage ratios which were less than or greater than the prespecified values were taken as zero and 1, respectively. Starting with the reduced generalized configuration, the search was renewed in the smaller parameter space. This stagewise search procedure was continued as long as the successive reduced configuration resulted in an improvement in the objective function. As pointed out by Hendry, Rudd, and Seader (*19*), this strategy may not lead to the global optimum although for the two cell case, treated in the sequel, the convergence has been verified by independent means.

TWO SPECIES, TWO CELL PROBLEM

For demonstration, consider first the case $n = m = 2$. This will permit evaluation of the efficacy of the integral method by comparison with the enumerated structures, in Fig. 1, optimized individually with respect to the mean residence times in cells I and II. Table 1 lists the numeric values of feed characteristics and the constraints on process design variables employed for this exercise. Objective functions identified by A and B were simply maximize recovery subject to constraint that grade does not fall below 35 and 75%, respectively. In addition a profit function C, given below, was also maximized.

$$C = P_F + (G - G_F)P_G \tag{24}$$

where P_F is the price of feed ore of grade G_F and P_G is the increase in price of the concentrate for every 1% improvement in grade.

Results are presented in Tables 2A, 2B, and 2C where initial guess, optimal configuration at first stage, and the final optimal structure synthesized from the reduced configuration are shown. The reduction was carried out by equating large and small linkage parameters (>0.8 and <0.2, respectively) with 1 and zero, respectively. As expected, in the first two cases optimum occurs at the grade constraint boundary, but depending on the grade stipulation, the structures of the optimal circuits are quite different. For objectives A and B the resulting configurations are

TABLE 1

Numeric Values Used in Computation of Design Parameters in Two Cell Circuit Problem

Feed composition, %	Constraints on process variables	Values of the parameters
Valuable = 22	$V \leq 20$ cu ft	$K_{val} = 1$/min
Gangue = 78	$d_p = 20\%$	$K_{gang} = 0.1$/min
	$0 \leq \lambda_1, \lambda_2 \leq 20$	$M_F = 25$ lb/min
		$P_F = 10.0$ units
		$P_G = 1.00$ units

TABLE 2A

Values of Designed Parameters for Stagewise Optimization for Two Cell, Two Species Problem for Objective *A*

Parameters	Initial value	Stage I	Stage II	Optimal values
δ_{10}	0.525	0.764	1.00	1.00
δ_{11}	0.150	0.067	0.00	0.00
δ_{12}	0.325	0.169	0.00	0.00
δ_{20}	0.717	0.00	0.00	0.00
δ_{21}	0.101	0.494	1.00	1.00
δ_{22}	0.182	0.506	0.00	0.00
β_{10}	0.346	0.184	0.00	0.00
β_{11}	0.185	0.122	0.00	0.00
β_{12}	0.469	0.694	1.00	1.00
β_{20}	0.661	0.900	1.00	1.00
β_{21}	0.152	0.00	0.00	0.00
β_{22}	0.187	0.100	0.00	0.00
δ_{F1}	0.254	0.00	0.00	0.00
δ_{F2}	0.746	1.00	1.00	1.00
λ_1	6.841	9.100	18.545	18.700
λ_2	5.238	2.015	3.945	3.730
V_1	—	8.08	11.90	12.51
V_2	—	6.59	7.48	7.48
Recovery	—	61.49	61.99	62.01
Grade	—	35.00	35.00	35.00

TABLE 2B

Values of Designed Parameters for Stagewise Optimization for Two Cell, Two Species Problem for Objective *B*

Parameter	Initial value	Stage I	Stage II	Optimal values
δ_{10}	0.352	0.00	0.00	0.00
δ_{11}	0.320	0.522	0.00	0.00
δ_{12}	0.328	0.478	1.00	1.00
δ_{20}	0.840	0.973	1.00	1.00
δ_{21}	0.088	0.012	0.00	0.00
δ_{22}	0.072	0.015	0.00	0.00
β_{10}	0.643	0.783	1.00	1.00
β_{11}	0.144	0.141	0.00	0.00
β_{12}	0.213	0.076	0.00	0.00
β_{20}	0.025	0.002	0.00	0.00
β_{21}	0.485	0.619	1.00	1.00
β_{22}	0.490	0.379	0.00	0.00
δ_{F1}	0.078	0.00	0.00	0.00
δ_{F2}	0.912	1.00	1.00	1.00
λ_1	1.093	0.996	0.658	0.658
λ_2	2.527	8.650	14.032	14.032
V_1	—	1.612	1.420	1.420
V_2	—	12.170	18.580	18.580
Recovery	—	22.350	24.867	24.867
Grade	—	35.00	35.00	35.00

TABLE 2C

Values of Designed Parameters for Stagewise Optimization for Two Cell, Two Species Problem for Objective C

Parameters	Initial value	Stage I	Stage II	Optimal values
δ_{10}	0. 518	0. 908	1. 00	1. 00
δ_{11}	0. 377	0. 088	0. 00	0. 00
δ_{12}	0. 105	0. 004	0. 00	0. 00
δ_{20}	0. 285	0. 006	0. 00	0. 00
δ_{21}	0. 237	0. 443	1. 00	1. 00
δ_{22}	0. 478	0. 551	0. 00	0. 00
β_{10}	0. 651	0. 001	0. 00	0. 00
β_{11}	0. 236	0. 325	0. 00	0. 00
β_{12}	0. 113	0. 674	1. 00	1. 00
β_{20}	0. 670	0. 880	1. 00	1. 00
β_{21}	0. 049	0. 002	0. 00	0. 00
β_{22}	0. 281	0. 118	0. 00	0. 00
δ_{F1}	0. 246	0. 271	0. 00	0. 00
δ_{F2}	0. 754	0. 729	1. 00	1. 00
λ_1	8. 948	13. 460	14. 816	14. 806
λ_2	4. 353	0. 715	0. 822	0. 822
V_1	—	16. 65	17. 31	17. 31
V_2	—	3. 33	2. 69	2. 69
Recovery	—	34. 69	33. 630	33. 645
Grade	—	57. 60	60. 70	60. 70
Price	—	395. 70	409. 76	409. 78

equivalent to configuration No. VI and V in Fig. 1 (cell number 1 in computed configuration refers to cell number 2 in Fig. 1 and vice-versa). The last column under each of the three objectives presents results of the trial and error approach obtained by optimizing individually the six configurations in Fig. 1 with respect to residence times only. The agreement provides a partial check that it is possible, at least in this case, to derive an optimal circuit starting from the generalized configuration in Fig. 2.

The two optimal circuits are intuitively reasonable. Thus when recovery is to be increased at some sacrifice of the grade, as in objective A, it is obvious that one of the cells must act as a scavenger [cell II in Fig. 1 (VI)]. On the other hand, in objective B high grade restriction makes it necessary that the rougher concentrate be refloated in cleaner cell [cell II, in Fig. 1 (V)].

THREE SPECIES, FOUR CELL PROBLEM

From the industrial flotation point of view, this is a more meaningful problem. The third particulate species is middlings in which the valuable mineral is locked in a gangue matrix. Table 3 gives the numeric values

TABLE 3

Numeric Values Used in Computation of Design Parameters in Four Cell, Three Species Problem

Feed composition, %	Constraints	Values of the parameters
Valuable = 15	$G = 35$	$K_{val} = 1$/min
Middling = 20	$0 \leq \lambda_j \leq 20, j = 1, 2, \ldots, 4$	$K_{mid} = 0.1$/min
Gangue = 65	$\bar{d}_p = 20$	$K_{gang} = 0.01$/min
	$V = 80$ cu ft	$M_F = 25$ lb/min

used for maximizing the recovery subject to a minimum of 35% grade. As shown in Fig. 3, the generalized structure has a rather large number of parameters, and attainment of convergence in a reasonable time is doubtful. Therefore, optimization was carried out in four stages. At the end of a stage, all split stream ratios $\leqq 0.05$ were set equal to zero and those $\geqslant 0.95$ were equated with 1. Each stage resulted in improvement of the recovery. The optimal structure is shown in Fig. 4; the linkages parmeters are given in parantheses. Table 4 gives the stagewise progress of the linkage and design parameters and the objective function.

It will be noted in Fig. 4 that in the optimal configuration, there is one split stream of value 0.17 only. In order to simplify the structure, it was decided to remove this stream and seek new parameters for best recovery. Results of this stage 5 computation, in column 7 of Table 4, show that the recovery has now fallen somewhat. Further reduction was carried out in stage 6 by eliminating cell No. IV altogether from the circuit, since its computed volume was comparatively small as compared to other cells. It will be seen from column 8 of Table 4 that the recovery has further decreased somewhat but the final circuit is now considerably simpler. The last two structures are shown in Fig. 5, with stage 5 structure in broken and stage 6 in solid lines. This example illustrates that, given the necessary data, it should be possible to incorporate capital investment and operational cost also in a more comprehensive profit function for an objective decision on choice of the most efficient circuit.

DISCUSSION

The main advantage of the integral approach over other methods (*20–22*) lies in the fact that it will give a split stream directly, if one exists as shown in Table 4, whereas in other approaches there is no

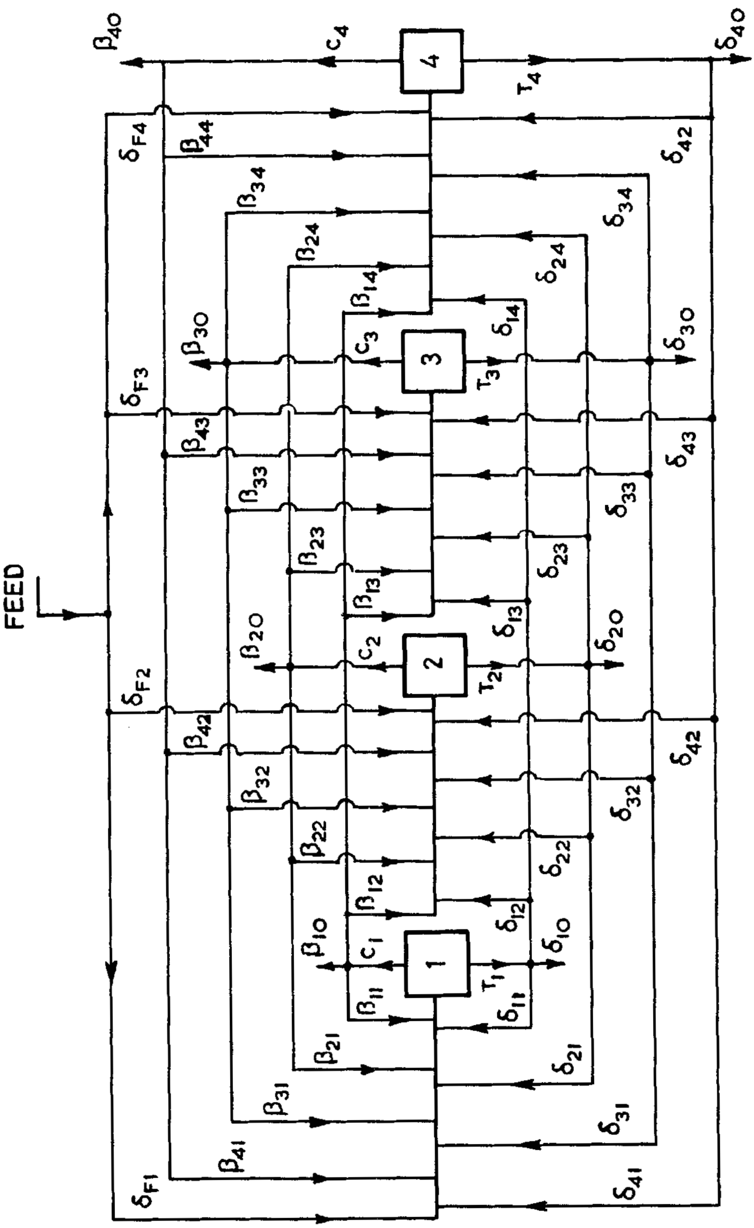

FIG. 3. Generalized configuration for four flotation cells.

TABLE 4

Values of Designed Parameters for Stagewise Optimization for Four Cell, Three Species Problem

Parameter	Initial value	Stage I	Stage II	Stage III	Stage IV	Stage V	Stage VI
δ_{10}	0.484	0.031	0.002	0.00	0.00	0.00	0.00
δ_{11}	0.046	0.068	0.031	0.00	0.00	0.00	0.00
δ_{12}	0.180	0.389	0.352	0.00	0.00	0.00	0.00
δ_{13}	0.166	0.403	0.615	1.00	1.00	1.00	1.00
δ_{14}	0.124	0.109	0.00	0.00	0.00	0.00	—
δ_{20}	0.541	0.049	0.016	0.002	0.00	0.00	0.298
δ_{21}	0.250	0.251	0.014	0.00	0.00	0.00	0.00
δ_{22}	0.130	0.367	0.238	0.743	0.654	0.680	0.702
δ_{23}	0.036	0.165	0.423	0.00	0.00	0.00	0.00
δ_{24}	0.043	0.168	0.309	0.255	0.346	0.320	—
δ_{30}	0.285	0.019	0.020	0.00	0.00	0.00	0.00
δ_{31}	0.183	0.00	0.00	0.00	0.00	0.00	0.00
δ_{32}	0.207	0.800	0.945	1.00	1.00	1.00	1.00
δ_{33}	0.099	0.143	0.035	0.00	0.00	0.00	0.00
δ_{34}	0.226	0.038	0.00	0.00	0.00	0.00	—
δ_{40}	0.507	0.403	0.543	0.982	1.00	1.00	—
δ_{41}	0.220	0.045	0.00	0.00	0.00	0.00	—
δ_{42}	0.033	0.072	0.00	0.00	0.00	0.00	—
δ_{43}	0.219	0.407	0.457	0.018	0.00	0.00	—
δ_{44}	0.021	0.073	0.00	0.00	0.00	0.00	—
β_{10}	0.371	0.736	0.658	1.00	1.00	1.00	1.00
β_{11}	0.156	0.003	0.00	0.00	0.00	0.00	0.00
β_{12}	0.244	0.018	0.00	0.00	0.00	0.00	0.00
β_{13}	0.061	0.239	0.342	0.00	0.00	0.00	0.00
β_{14}	0.168	0.004	0.00	0.00	0.00	0.00	—
β_{20}	0.414	0.990	1.00	1.00	1.00	1.00	1.00
β_{21}	0.210	0.007	0.00	0.00	0.00	0.00	0.00
β_{22}	0.245	0.00	0.00	0.00	0.00	0.00	0.00
β_{23}	0.084	0.00	0.00	0.00	0.00	0.00	0.00
β_{24}	0.047	0.003	0.00	0.00	0.00	0.00	—
β_{30}	0.551	0.582	1.00	1.00	1.00	1.00	1.00
β_{31}	0.021	0.077	0.00	0.00	0.00	0.00	0.00
β_{32}	0.205	0.194	0.00	0.00	0.00	0.00	0.00
β_{33}	0.406	0.014	0.00	0.00	0.00	0.00	0.00
β_{34}	0.177	0.133	0.00	0.00	0.00	0.00	—
β_{40}	0.323	0.105	0.81	0.030	0.00	0.00	—
β_{41}	0.150	0.359	0.090	0.702	0.170	0.00	—
β_{42}	0.058	0.227	0.089	0.268	0.830	1.00	—
β_{43}	0.249	0.308	0.00	0.00	0.00	0.00	—
β_{44}	0.220	0.001	0.010	0.00	0.00	0.00	—
δ_{F1}	0.080	0.279	0.637	0.577	1.00	1.00	1.00

(continued)

TABLE 4 (*continued*)

Parameter	Initial value	Stage I	Stage II	Stage III	Stage IV	Stage V	Stage VI
δ_{F2}	0.004	0.550	0.362	0.423	0.00	0.00	0.00
δ_{F3}	0.045	0.170	0.00	0.00	0.00	0.00	0.00
δ_{F4}	0.871	0.001	0.001	0.00	0.00	0.00	—
λ_1	4.580	13.015	11.707	14.88	18.09	19.89	20.0
λ_2	4.390	15.028	14.206	19.98	19.42	18.81	18.29
λ_3	0.196	5.077	7.107	15.19	19.02	16.54	16.88
λ_4	1.61	4.332	14.803	14.88	8.41	5.93	—
V_1	—	20.42	10.23	10.67	20.18	21.60	21.69
V_2	—	45.73	42.96	47.95	31.14	31.84	31.42
V_3	—	9.78	14.99	8.85	16.43	14.31	14.54
V_4	—	3.41	11.80	7.77	4.28	3.02	—
Recovery	12.80	69.08	70.27	70.72	71.18	71.08	70.93
Grade	79.40	35.00	35.01	35.00	35.00	35.00	35.00

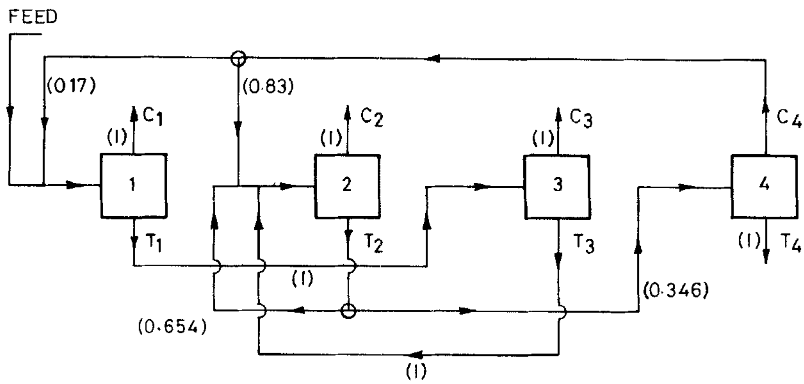

FIG. 4. Optimal configuration for three species, four cell problem.

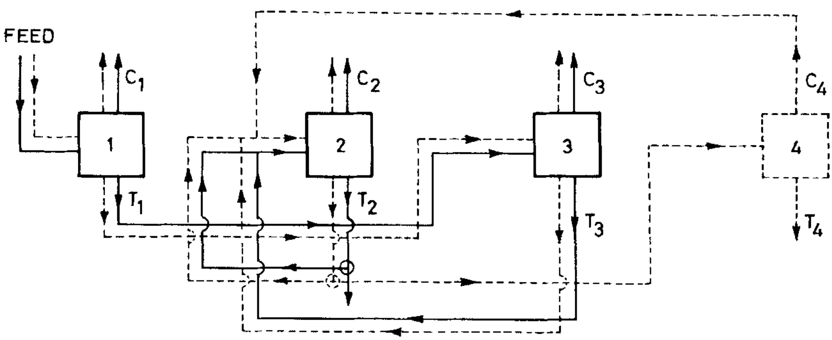

FIG. 5. Suboptimal configuration for three species, four cell problem.

way of obtaining split streams even if they are necessary for the optimal structure. The integral method and the trial and error approach lead to identical configurations with almost equal design parameters for objectives A, B and C.

It should be stressed that the examples chosen in this investigation were deliberately kept simple in order to focus attention on the nature of the problem and its possible solution. In theory at least, there is no reason why more general processes cannot be synthesized. A battery of flotation cells, commonly used in industry, can be modeled as x number of perfect mixtures in series, and for equal residence time Eq. (1) becomes

$$T(K) = M_F(K)\left[\frac{1}{1 + K\lambda}\right]^x \tag{25}$$

Distributed size feed can also be handled if it is assumed, as a crude approximation, that for sufficiently dilute suspensions, the sojourn time of solids is a function of particle size but is independent of particulate environment. Equation (2) for particles of size r can then be written as

$$T(K_r) = M_F(K_r)\left[\frac{1}{1 + K_r\lambda_r}\right] \tag{26}$$

Now the design parameters to be optimized are the constants $a, b, \ldots,$ in an empirical expression relating λ to size r

$$\lambda_r = \lambda(r, a, b, \ldots) \tag{27}$$

Experimental data of Woodbrun et al. (9) shows that the following relationship is perhaps sufficient for this purpose:

$$\lambda_r = a + br + cr^2 \tag{28}$$

The main drawback in the integral approach, as indicated earlier, is in the optimization step which becomes even more acute as the parameter space increases. Recently, the authors have come across a more efficient hyperconical random search method (*23*) which may prove to be more suitable for this kind of problem.

In conclusion, the optimal synthesis of the flotation circuit and its design variables are highly sensitive to the nature of feed material, number of cells, and the objective to be maximized. The integral approach provides a formal systematic technique for making decisions for this purpose. It will be appreciated that the synthesis of split stream circuits shown here would have been highly unlikely by empirical arguments or an heuristic approach only. In view of the large number of parameters involved, it may not be possible to guarantee the global optimum. But on the other hand, realization of a large number of local optima can be exploited to choose a circuit which is both reasonably efficient process-wise and simple in design, as shown in the four cell example.

SYMBOLS

a, b, c	parameters in empirical expression relating mean residence time to particle size
$C(K)$	mass flow rate of particles K in concentrate stream
$C_{j,k}$	mass flow rate of jth species in concentrate stream from kth cell
d	density of particulate solids
d_p	pulp density
$\bar{d}_p$	limiting value of pulp density
$F_{j,i}$	total feed flow rate of jth species to ith cell
G	grade of concentrate
$\bar{G}$	limiting value of grade
G_F	grade of feed ore
H_i	hold up of solids in ith cell
K	flotation rate constant
K_j	flotation rate constant of jth species
K_{gang}	flotation rate constant of gangue
K_{mid}	flotation rate constant of middlings
K_{val}	flotation rate constant of valuable
m	number of cells in circuit
$M_F(K)$	mass flow rate of particles K in new feed
n	number of particulate species in feed
P_F	price of feed ore
P_G	increase in price of concentrate for every 1 % improvement in grade
r	particle size
$T(K)$	mass flow rate of particles K in tailings stream
$T_{j,k}$	mass flow rate of jth species in tailings stream from kth cell
V	total volume of cells in circuit
V_i	volume of ith cell
w_j	fractional valuable mineral content in jth species
x	number of cells in a battery of flotation cells
α_{ji}	defined by Eq. (4)
β_{i0}	fraction of concentrate flow from ith cell leaving the circuit
β_{ki}	fraction of concentrate flow from k to ith cell

δ_{i0} fraction of tailings flow from ith cell leaving the circuit
δ_{ki} fraction of tailings flow k to ith cell
δ_{Fi} fraction of new feed to ith cell
λ mean residence time of solid particles in the cell
$\bar{\lambda}$ limiting value of mean residence time of solid particles in the cell
λ_i mean residence time of solid particles in ith cell

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